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STOCHASTIC DIFFERENTIAL EQUATIONS

Short PhD course (Barcelona, January 2014)

Course structure: Three 2-hour lectures

Aim: The aim of this course is to develop the theory of stochastic differential equations and study certain path properties of diffusion processes using tools from semimartingale theory. Lecture 1 will be introductory while Lectures 2 and 3 will cover recent research topics.

Required knowledge: Basics of stochastic processes (familiarity with filtrations, processes in discrete and continuous time, Brownian motion, basic properties of martingales and Markov processes), quadratic variation and stochastic calculus (stochastic integrals, Itô's formula).

Lecture 1: Background.

- Local time for continuous semimartingales, Itô-Tanaka formula, random time-change of stochastic integrals.
- The fundamental theorems: Dambis-Dubins-Schwartz theorem, Girsanov's theorem, martingale representation theorem.
- Overview of SDE theory: strong and weak solutions of SDEs, uniqueness in law, pathwise uniqueness, existence theorems (Itô, Skorokhod), Yamada-Watanabe theorem, examples and counterexamples.

Lecture 2: Characterisations of the martingale property of stochastic exponentials.

- Martingale problems and weak (possibly exiting) solutions of SDEs.
- Engelbert-Schmidt construction of weak solutions and uniqueness in law.
- Generalised densities and Jessen's theorem.
- Separating times and the absolute continuity and singularity of probability measures on a filtered path space.
- Characterisation of the martingale property of diffusion-based exponential local martingales.

Lecture 3: Additive functionals of diffusion processes.

- Path properties and the continuity of the local time random filed of solutions of SDEs.
- Convergence of additive functionals of diffusions (two proofs will be discussed: one based on the Williams theorem and Bessel processes and the other using the Ray-Knight Theorem).
- The loss of the semimartingale property of diffusion processes at the hitting time of a level.

References

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[5] A. Mijatović and M. Urusov. Convergence of Integral Functionals of One-Dimensional Diffusions. *Electronic Communications in Probability* (2012).

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[8] Protter, Ph., 2005, *Stochastic Integration and differential equations*, Vol. 21 of *Applications of Mathematics, Stochastic Modelling and Applied Probability*, Springer.

[9] Revuz, D., Yor M., 1999, *Continuous martingales and Brownian motion*, 3rd edn, Springer-Verlag, Berlin Heidelberg.