Per cognitionem visualem
The Visualization of Cognitive and Natural Processes in the Middle Ages

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Per cognitionem visualem. The Visualization of Cognitive and Natural Processes in the Middle Ages

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Edited by
José Higuera Rubio

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FOREWORD

JOSÉ HIGUERA

This volume offers an overview of medieval representations of knowledge under a philosophical perspective. The study of imagery examples and thinking practices introduced by a diverse scholars seeks to explore representation uses, analytical approaches on perception, and the context's transmission of visual frameworks. Incidentally, contemporary studies on diagrammatic thought systematize those complex relations into scientific practices, perceptual patterns, and contextual insights. The undeniable value of visual tools linked with knowledge spreading and learning practices during the Middle Ages assemble manuscript traditions, intellectual practices, and philosophical debates. Visual tools, frequently appearing in Medieval manuscripts, have often been considered "illustrative material" intended to facilitate textual comprehension. However, these visual frameworks offer something more than a straightforward correspondence between a conceptual interpretation and its figurative depiction. They are, in fact, key to understanding the methods of acquiring and shaping knowledge through visual frameworks with didactical, disputational or heuristic purposes.

In four sections, this volume attempts to tackle those complex questions, gathering contributions presented to the XXV SIEPM Colloquium at Porto–Instituto de Filosofia, delayed to June of 2021 as a consequence of COVID-19's social contact restrictions. The first section points out the overlapping of tree-diagrams in dialectical representations, biblical narratives, and Lullian art. The second introduces the reading patterns of prognosis through astrological and geomantical figures. The third revisits the accuracy of language concerning perceptual and metaphysical content. Finally, the last section deals with the variables of visual properties around mirrors and physical motion.

The way to attempt this quest certainly matched very well with the intellectual environment generously offered by the Instituto de Filosofia (Universidade do Porto/IF-FLUP) for organizing the XXV SIEPM Colloquium. Actually, the Instituto de Filosofia had been shaped from

the start, in 1997, by the only Iberian research institution focused on the medieval history of thought: the Gabinete de Filosofia Medieval (GFM), founded by Prof. Maria Cândida Pacheco in 1987. As SIEPM's assessor and vice-president (1992-2002), author of many studies, and a committed promoter of prolific researching careers, Prof. Pacheco – who passed away in 2020 – deserves fervent recognition in this volume, with strong grateful feelings from the community of medieval philosophy studies. The editor would also like to thank the SIEPM Bureau for its support and engaged participation in this virtual colloquium, especially SIEPM's President, Prof. Maarten Hoenen, and SIEPM's General Editor, Prof. Alessandra Beccarisi, for supporting the publication of this volume.

Universidad Nacional de Educación a Distancia-UNED

Ramon Llull and the *Ars Diagrammatica*

Albert Soler

Ramon Llull (ca. 1232-1316) was a Majorcan layman without formal studies who, towards the middle of his life (ca. 1263), felt a supernatural need to pious conversion, a fact not uncommon in the spirituality of the 12th and 13th centuries. The alteration of his life, in this case, included the project of writing "unum librum, meliorem de mundo, contra errores infidelium," according to his autobiography.[1] This book reached its first achievement in the monumental *Llibre de contemplació en Déu* [*Book of Contemplation on God*], but as soon as this work was finished, around 1274, Ramon received a divine "illumination" on the mountain of Randa (Majorca) by which God infuses the "formam et modum faciendi librum."[2] From this revelation, he wrote more than 260 works. It is striking, however, that in order to remain faithful to what was revealed to him, Llull constantly felt compelled to conceptually and formally innovate in his books.

The product of Randa's illumination was the Art, a system for thought foundation, for the discovery and communication of truth (Christian Truth, of course), free from the authority's arguments (including the authority of the Holy Scriptures), suitable for Muslims, Christians and Jews, and therefore with an apologetic purpose, above all.[3] Lullian Art had several versions in an attempt to make it more and

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[1] Raimundus Lullus, *Vita coaetanea*, ed. H. Harada (CCCM 34), Turnhout 1980, 275. In addition to the archival documents and data provided by the colophons of his works, we have an almost autobiography, the *Vita magistri Raimundi*, also known as *Vita coaetanea*, dictated by Ramon to a monk of the Carthusian monastery of Vauvert, near Paris, in 1311.

[2] Raimundus Lullus, *Vita*, ed. H. Harada, 280: "subito Dominus illustrauit mentem suam, dans eidem formam et modum faciendi librum."

[3] For an introduction and user manual of the Art, see A. Bonner, *The Art and Logic of Ramon Llull. A User's Guide* (Studien und Texte zur Geistesgeschichte des Mittelalters 95), Leiden-Boston 2007.

more accessible and understandable.[4] Through of his Art, Ramon will composed endless applications to the most varied sciences (medicine, law, theology, philosophy, geometry, astronomy...) in the most diverse genres and formats (treatises, sermons, novels, poetry, proverbs...), until reaching the number of works mentioned above.

It becomes clear, therefore, that Ramon's illumination had a creative impulse: it was not a closed revelation that had provided a way to make that book, the best in the world, and that it should be stated and not modified, if you wanted to be faithful to the order received. On the contrary, the different versions of Art show plurality of forms and dispositions. This is so because the illustration did not consist of a message but, as has already been said, of a system. A supernatural message is always problematic because the Revelation has already been given by Jesus Christ once and for all and, therefore, when someone claims to have received a new revelation, they should clarify what kind of nuance it brings regarding the original one. However, what was revealed to Ramon Llull in Randa was a new persuasion system regarding the Christian revelation truth, not a new truth. That is why the form has a substantial importance in the development of Llull's work and his concern about the way to express it, among other things, by the use of a wide range of visual resources in his works: text page, layout formats, diagrams, figures, and, within the latter, circular figures, trees, tables... and, more rarely, some symbolic representation.[5]

Llull expects readers to adopt a proactive attitude towards his works. Rather than telling them what to think, he sees them as means to develop their intellectual capacities. The Majorcan author even considers very different degrees of training and non identical intellectual starting points. Adapting to the diversity of audiences and circumstances, he

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[4] From the initial *Ars compendiosa inveniendi veritatem*, composed as a consequence of illumination (ca. 1274), and *Ars demonstrativa* (ca. 1283), which are the core of the first phase of the Art. Until *Ars generalis ultima* (1305-1308), which closes the tour together with its abbreviated version, *Ars brevis* (1308), which, with the preceding *Ars inventiva veritatis* and *Ars amativa* (both from 1290) and the *Tabula generalis* (1293-94), make up the main versions of the second phase of the Art.

[5] M. W. Evans "The geometry of the mind", in *Architectural Association Quarterly* 12.4 (1980), speaks of four "main types of visual aid: the typographic, the stemmatic, the geometric and the emblematic" (§ 3.2).

writes his work in Catalan, Latin or Arabic;[6] and, for the same reason, he writes abstract treatises, romans inspired by the chivalric tradition or didactic poetry. In the background, there is always a method, the Art, which ensures the correct orientation of what is writing and what is reading.

A paradigmatic example (one among many) of Llull's desire to actively involve the reader in his work is the renowned ending of the *Llibre del gentil e dels tres savis* [*Book of the Gentile and the Three Wise Men*] (1274-1276). In the narrative fiction, Ramon envisages a gentile, someone who is pagan and ignores monotheism, improving his knowledge of faith through the three wise men's teachings, the three guides: a Jew, a Christian and a Muslim. After elucidating the disciple about the existence of a single God, about the creation and about the resurrection (truths that the three wise men admit), each one presents his own religion in a kind and respectful way so the gentile should choose one which he believes to be true. It is relevant how Ramon carries out a systematic exposition of the principles of the Mosaic law and Islam, with a responsible and sufficient knowledge of both and without clues that discrediting. Once the discussion is over, the gentile affirms that he already knows which of the three religions is the true one, but the wise men beg him not to tell them his choice because it would condition the continuation of their discussion. The book ends without explicitly declaring the protagonist's decision, so the reader, if he wants to access the solution, has no other choice but to emulate the gentile and start thinking in order to arrive at the truth on his own.

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[6] Llull wrote a substantial part of theological or scientific works in his mother tongue, Catalan; at the same time, he encouraged the circulation of these works in other Romance languages. It is not an obstacle for him to write a considerable number of titles in Latin, or for some of these titles to have a double version, Catalan and Latin. Whatever the original language of a Lullian work, they are all part of the same intellectual project developed by someone who had an anomalous access to the world of knowledge and who organized the production and dissemination of his writings in an unconventional way. Although there is no doubt that they existed, we have not preserved Lullian works in Arabic. In L. BADIA, J. SANTANACH and A. SOLER, *Ramon Llull as a Vernacular Writer: Communicating a New Kind of Knowledge*, London 2016, it is proposed that all the peculiarity of the "Ramon Llull" phenomenon be interpreted as a manifestation of the process of vernacularization of knowledge which occurs in the fall of the Middle Ages.

This is an apologetic work, designed to demonstrate the effectiveness of Llull's method in a discussion about the truth or falsity of the three laws or book's religions. The reader must be a man of good faith, willing to follow the 'conditions' inscribed in the 'flowers' of the five symbolic 'trees'. They are binary combinations between the God's attributes, the virtues, and vices, which are associated by concordance or by contrariety. If these conditions of the discursive game are admitted, the incontrovertible triumph of Christianity is automatically imposed, without any participant in the debate feeling undermined or threatened. It is the victory of intelligence over the authorities. As A. Bonner has pointed out in his Art "user's guide":

> Perhaps the greatest difference between the Art and contemporary treatises, however, is that whereas the latter are primarily directed at offering solutions to problems or clarifications on points of doctrine, Llull's Art offers a method for arriving there, and if it gives solutions and clarifications – which it of course does – they are presented as inevitable by-products of that method.[7]

If the Lullian Art is capable of producing such a result, it is undoubtedly because it is an "Ars Diagrammatica". Thus, it is provided with a variety of visual resources that allow the reader to learn reasoning through an efficient way to reach the truth, and fully develop their faculties. In this paper, I propose to carry out a brief overview of the various types of visual aids used by Llull in his extensive work. Starting with the text's layout resources, passing through the diagrams themselves and a brief reference about the few symbolic representations. Finally, we will have a broader view of Llull's Art as diagrammatic reasoning.[8]

Layout of the text

Llull accomplished several versions of his Art while, at the same time, being the author of a particular *ordinatio* of the text which, in taking shape in the space of the page and the book, organizes and conducts mental reasoning[9].

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[7]   BONNER, *The Art*, 298.

[8]   In recent years, together with Anthony Bonner, we have developed a line of research on visual resources in Llull's opus that has given rise to several articles cited in the notes and of which this work is a synthesis.

[9]   J. P. GUMBERT "Le texte intellectuel, sa forme physique et les termes du metier", in

In the first version of the Art, *Ars compendiosa inveniendi veritatem* (ca. 1274, from now on *ArsComp*) Llull adopts layout formats that he will substantially modify in the following one, *Ars demonstrativa* (ca. 1283, from now on *ArsDem*), and will modify again in the drastic rethinking of the system of *Ars inventiva veritatis* (1290, from now on *ArsInv*). In the case of *ArsComp* and *ArsDem*, the arrangement and organization of the text has much to do with an extensive apparatus of graphic figures. Their components drawn in various colors have various fixed circular figures (*rotae*), diverse adjacency matrices that derive from those. There are also revolving circular figures – volvelles –, tables and an infinity of compartments called *camerae* in Latin.

In *ArsComp*, the written text is presented as an annex of graphic apparatus. Thus, we read in the prologue of the work:

> Haec *Compendiosa ars inveniendi veritatem* dividitur in quinque figures, quae sunt A S T V X. Et mediantibus istis quinque potest quis invenire veritatem sub compendio, et contemplando Deum virtutibus adherere et a se vitia extirpare.[10]

The work is not divided into chapters or parts but is structured around five circular figures (*rotae*). Next, five sections are presented, corresponding to the five *rotae*, each of one beginning in the same way "A [or S, T, V, X] ponimus quod sit..." The corresponding figure heads over the page, and is surrounded by a non-descriptive text that introduces general considerations about it. Up to a certain point, the image could be considered sufficient: the writing shows itself as a figure's gloss.

Actes du colloque Terminologie de la vie intellectuelle au moyen age. Leyde - La Haye 20-21 September 1985, ed. O. Weijers, Turnhout 1988, 96-104, distinguishes between intellectual text and material text (the physical form that shapes the former) and refers to the relationships of proximity or distance that are established between one and the other.

[10] A. Bonner and A. Soler, "La *mise en texte* de la primer versió de l'Art: noves formes per a nous continguts", in *Studia lulliana* 47 (2007), 36.

Fig. 1. Vat. lat. 5112, f. 5va. Section on the figure T of Art (*rota*)
(ArsComp)

The importance of the figures in *ArsComp*'s *mise en texte* highlights
the relevance that Llull gave to them already in the first version of Art.
They bring together all its components, which are general in the entire
system, and allowed its combinatorial mechanisms.

Fig. 2. Vat. lat. 5112, f. 6r at the bottom, the *Elemental Figure* (*in table form*) and, at the top, the Third Figure in adjacency matrix form T of ArsComp

The first presentation of the Lullian system could cause a deep sense of strangeness in a medieval reader habituated to more conventional texts in which graphics are mere illustrations of written discourse with pre-eminence and driving the reasoning's advancement. In *ArsComp*,

the relationship established between text and image and the circularity of the discourse that develops from this relationship pose important challenges to the reader. For example, in the presentation of Figure A, it speaks of S, T, V, X, Y, Z when the reader still has not the remotest idea of what those letters mean. It is only when this prologue on the five figures is read that the reader feels the need to go back to the beginning to understand what is explained in such a peculiar way.[11]

All these problems are solved in *ArsDem* thanks to a more linear organization of discourse. In the first place, the list of figures, which has been considerably expanded, offered at the beginning of the work, in a quire, he put together all of these except the adjacency matrices, which continue to be integrated into the text. Second, a conceptual alphabet is established at the beginning of the discourse:

> Quoniam haec *Ars demonstratiua* sequitur regulam *Artis compendiosae inueniendi ueritatem*, nos ideo alphabetum illius artis accipimus ponentes et ordinantes ipsum in hac arte; quod siquidem alphabetum tale est: A ponitur esse Deus. B memoria recolens. C intellectus intelligens [...][12]

Third, each figure is introduced and described in detail in the text and in those descriptions references are made to the initial on the graphic images. Thus, Figure T has a formal description that is absent in *ArsComp*:

> Prima enim circularis est, ex quinque triangulis uariorum colorum composita, prout apparet in principiis huius artis.[13] (1) Quorum triangulorum unus liuidus est, significans Deum in uno angulo, et creaturam in alio, atque in alio angulo operationem. (2) Et alius triangulus uiridis est, significans in uno angulo differentiam, et in alio concordantiam, atque in alio contrarietatem. (3) Alius uero triangulus rubeus est, significans in uno angulo principium, in alio medium, et in alio finem. (4) Alius etiam triangulus croceus est, significans in uno angulo maioritatem, in alio aequalitatem, atque in alio minoritatem. (5) Atque alius triangulus niger est, continens in uno angulo affirmationem, in alio dubitationem, in alio negationem, prout apparet manifeste in principii huius artis.[14]

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[11] For a detailed analysis of *ArsComp* layout issue, see Bonner and Soler, "La mise en texte".

[12] Raimundus Lullus, *Ars demonstrativa*, ed. J. E. Rubio Albarracín (CCCM 213), Turnhout 2007, 4.

[13] It refers to the quire with the graphic images that heads the work in the manuscripts.

[14] Lullus, *Ars demonstrativa*, ed. Rubio, 17

Fig. 3. Kues, Bernkastel-Kues, Nikolaus Hospital, ms. 83, f. 1v Figures S, T, Y, A, Z, V, X of *ArsDem*

The first clash with real audiences probably prompted a profound re-organization in the way the Lullian Art was presented, an aspect that is reflected in the arrangement of text and figures in the codices, in order to achieve a more efficient transmission of information.

On the other hand, in *ArsDem*, the visual resource of the *came-rae* gains special strength, that is, the compartments with letters – or terms – that come from combinations of principles of the Art. They are used more extensively and systematically in *ArsDem* than in *Ars-*

Comp or any other version of Art. In the manuscript – 220h of the Martinus-Bibliothek of Mainz – copied by Guillem Pagès – a scribe in the service of Llull for more than twenty-five years, from whom we have preserved six codices that have a special value for the study of the original formats[15] of Llull's works – all the text of the compartments was made by copying the terms in black ink and tracing the box frame in red ink, so that they stand out clearly on the page.[16]
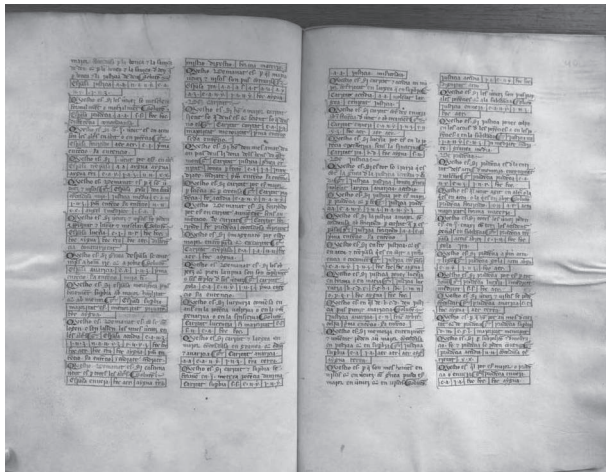

Fig. 4. Mainz, Martinus-Bibliothek, ms. 220h, f. 39v-40r. Prominence of the boxes in the fourth distinction, "De quaestionibus", of *ArsDem*

The next version of the Art, *ArsInv* (1290), represents a crucial moment in the development of the Lullian system because it proposes a drastic reformulation, undoubtedly due to the difficulties that Ramon experi-

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[15] For a comprehensive study of the codices copied by Guillem Pagès, see A. Soler, "Estudi històric i codicològic dels manuscrits lul·lians copiats per Guillem Pagès (ca. 1274-1301)", in *Arxiu de textos Catalans Antics* 25 (2006), 229-266.

[16] For a study of the visual resources in *ArsDem* see A. Soler, "Els recursos visuals a l'*Art demonstrativa* de Ramon Llull: impaginació i figures gràfiques", in *Llengua & Literatura* 32 (2022).

enced in getting his system recognized. The *Vita magistri Raimundi* points out Ramon's motivation to prompt a completely remake of his system due the unfavorable reception of *Compendium seu commentum Artis demonstrativae* in Paris. A work that, as its name indicates, depends on *ArsDem*:

> Veniens ergo Raimundus Parisius tempore cancellarii Bertoldi, legit ibidem in aula sua *Commentum Artis generalis* de speciali praecepto praedicti cancellarii. Perlectoque Parisius illo *Commento*, ac ibidem uiso modo scholarium, ad Montem rediit Pessulanum. Vbi de nouo fecit et legit etiam librum ipsum uocans eundem *Artem ueritatis inuentiuam*; ponendo in ipso libro, nec non et in omnibus aliis libris, quos ex tunc fecit, quattuor tantum figuras, resecatis seu potius dissimulatis propter fragilitatem humani intellectus, quam fuerat expertus Parisius, duodecim figuris ex sexdecim, quae prius erant in Arte sua.[17]

In the eleventh miniature that illustrates the *Vita magistri Raimundi*, in his *Breviculum*, Thomas Le Myésier, Llull's most outstanding disciple in Paris, asks the master permission to summarize his vast work and gives a view that possibly sums up that of the Parisian academic environment. He does this partly because:

> alleviare studium et fatigationem oculorum, confusionem significatorum alphabeti *demonstrativae Artis* et sexdecim eiusdem Artis figuris, quae confundunt intellectum.[18]

The main problem identified in this quoting, about the use of *ArsDem*, is the abundance of alphabetic figures symbols in the Art. The challenge was to simplify and to organize those materials in such way that would be less innovative and apparently unique to appeal to the academic habits of reading and reasoning. For readers, the most important drawback of *ArsComp* and *ArsDem* was Ramon's use of artificial language, based on alphabetic symbols organized in diagrams. A language that he wanted to be completely intellectual and was far removed from natural language. *ArsInv*, on the contrary, will develop a discourse built from ontological definitions that allow arguments to be generated through propositions that run as axioms, something that was much
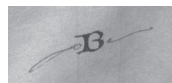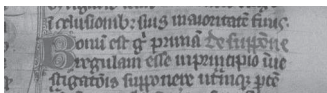
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[17] RAIMUNDUS LULLUS, *Vita*, ed. HARADA, 283.

[18] THOMAS MIGERIUS, *Breviculum seu Electorium parvum*, ed. Ch. LOHR, Th. PINDL-BÜCHEL y W. BÜCHEL (CCCM 77), Turnhout 1990, 45. This is a deluxe manuscript version of the *Vita magistri Raimundi* dedicated to the queen of France enriched with twelve magnificent illustrations made in 1325. Thomas Le Myésier, Canon of Arras, was court physician.
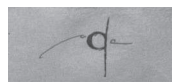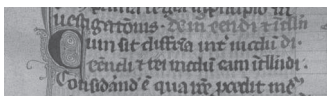
more familiar to contemporary readers. In this sense, the reduction of the number diagrams from *ArsDem* to *ArsInv* – from twelve to four, which have such a secondary role that makes them superfluous – is quite meaningful[19].

We have preserved an *ArsInv* manuscript related directly to Llull. It shows a layout project designed by him, with the help collaborators and copyists, for the new version of the Art. The Munich codex, Bayerische Staatsbibliothek, Clm. 10501, in which the decoration and structure are closely related to the writing of the text, shows that the organizing and retrieving of information through layout becomes a powerful visual resource. Thus, in the third and fourth distinctions of the work, with the intention of making the text more accessible, each chapter begins (see the table below, first and second column) with a word whose initial letter corresponds to the letters of the Art's Alphabet B-K. A letter that later appears in the running titles of the folios (third column). Let us see, as an example, the case of the first four chapters of the third distinction, on the Rules of the Art:
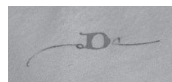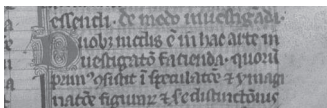
Bonum est igitur primam regulam esse in principio...		
Cum sit differentia inter modum essendi rei et modum eam intelligendi...		
Duobus modis est in hac arte inuestigatio facienda, quorum primus consistit in speculatione...		
Ex generali omnino generali et speciali tertium constituitur...		
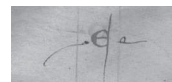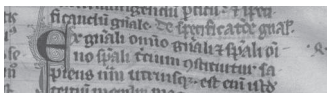
The function of the current titles is even more important in the fourth

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[19] A. Bonner, "Supplement", in *The Art and Logic of Ramon Llull. A User's Guide (Studien und Texte zur Geistesgeschichte des Mittelalters 95)*, Leiden-Boston 2007, 31; electronic publication <https://www.ub.edu/llulldb/bibliografia/9319>

distinction of *ArsInv* "De quaestionibus" in which nine questions corresponding to the letters A-K are resolved in three different ways corresponding to the first distinction (on the figures), the second (on the conditions) and the third (on the rules). In the manuscript, this structure deserves a singular layout treatment that, at the same time, outlines the text. Readers guided by it can spot specific questions or its resolution. The first mode is always preceded by a pen-flourished initial of a two lined sheet, parted letter[20], and cascade, to resemble the letters of questions B-K: B for the first, C for the second, etc.

Instead, the second and third modes are marked only with a cascade and pen-flourished initial, the letter no longer corresponding to Alphabet B-K.

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[20] Parted letter is that "Lettre dont les parties pleines sont subdivisées dans leur longueur, selon une ligne ondulée, brisée ou échancrée, en deux moitiés dont chacune est peinte d'une couleur différente", D. MUZZERELLE, *Vocabulaire codicologique*, s.v. "Lettre émanchée", electronic resource, <http://www.palaeographia.org/vocabulaire/vocab.htm> (accessed 14/01/2022). It should be noted that the parted letters were more laborious to make – they were charged at a different, higher price than the other initials, only pen-flourished initials – and, therefore, indicated a higher level in the text order.

The running title of the quoted folios points out the combinatory solutions according to the first mode with the same initial letter B-K to list the nine questions. The running titles of the second mode carry two letters in the running title; and the third has three letters; always with the letter that corresponds to each of the nine questions.
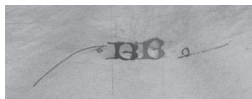
In short: the parted letter that opens the first mode of each question and the running title that indicates the folios where the solution is presented, act as markers that allow the reader to identify the question amongst the nine and to locate it if necessary. On the other hand, the style of the running titles, which mark with one, two or three letters B-K each mode of each question, identifies which of the three modes the reader is accessing or how to locate it.

The interrelationship observed between the writing of the work and the material realization of the copy, in the manuscript Clm. 10501, enables the study of aid elements that academic books had already fully incorporated in the 13th century to define the work *ordinatio*: running titles, rubrics, hierarchical initials, paragraph marks (Pied-de-mouche), numbering. In this sense, the written page of the mentioned manuscript is not a simple text record, but rather the visual representation of its organization.[21]

Graphic devices: graphs, hypergraphs, and tables

In the previous section, I have referred to different types of graphic representations of Lullian Art: *rotae*, volvelles, *camerae*, adjacency matrices, tables. However, more important than establishing a formal typology of these devices is to know how they work, the use that the system gives to elements and the reason for that use.[22]

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[21] For a study of the visual resources in the manuscript Clm 10501, see A. Soler, "Raonament intel·lectual i materialitat del text: el cas de l'*Ars inventiva veritatis* de Ramon Llull", in *Comprendre* 21,2 (2019), 57-73.

[22] For a detailed study of the nature and function of figures in Art, see A. Bonner

The main devices are the Alphabet, the Figures, and the Table. The Alphabet assigns letters to various concepts of the Art. In *ArsDem*, the twenty-three letters of the medieval Latin alphabet represent a complex display of concepts and figures. From the reformulation of *ArsInv*, in which the number of principles is reduced from sixteen to nine, the letters from B to K represent the six concepts of each series of the components of the Art.

Figures are used to gather various Art concepts into homogeneous sequences. Most of the figures in *ArsComp* and *ArsDem* are graphs, representations of binary relations established between elements in a set. In graph theory, the elements are called vertices and the lines that join them, edges. In the case of Art, graphs show connections between pairs of concepts. Let us see Figure A of *ArsDem,* which brings together the sixteen divine attributes or dignities:
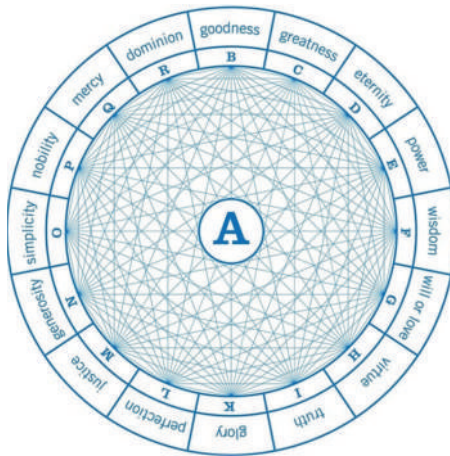
Fig. 5. Figure A of *ArsDem*

In this figure, all the vertices are connected to each other. In this way, it is indicated that divine attributes fully agree with each other. The graph itself is in the network of interior lines; the external *rota* serves only to contain the set of divine attributes.

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and A. Soler, "Les figures lul·lianes: la seva naturalesa i la seva funció com a raonament diagramàtic", in *Studia lulliana* 55 (2015), 3-30.

The same figure, called The First Figure, in *ArsInv* reformulation no longer has edges connecting the vertices and therefore it is no longer a graph:
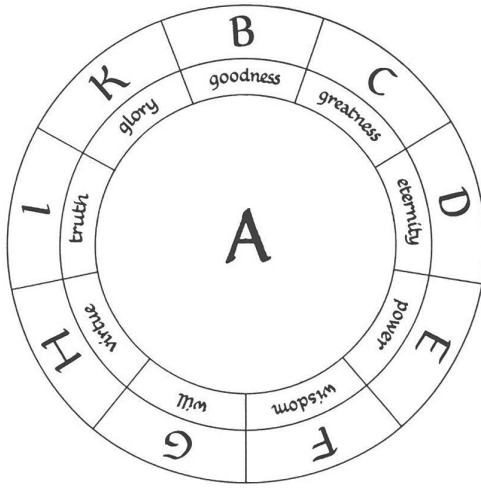
Fig. 6. First Figure of *ArsInv*

ArsInv does not work through the binary combination of principles, which is why the edges of the figure disappear. The use of a *rota* to collect the set of principles is only a formal vestige of the previous figure: it could have been replaced by a list in a table form.

In *ArsComp* or *ArsDem*, the representation of the pairs of concepts connected by lines in Figure A can take a tabular form that, in graph theory, is called an adjacency matrix. Normally, a matrix that includes all possible combinations would be a square. However, in the Art, the distinction between the pair "goodness greatness" from the pair "greatness goodness" is useless, and therefore only half of each matrix will be represented, which will give rise to a triangular shape, like the one in Second Figure of Figure A:
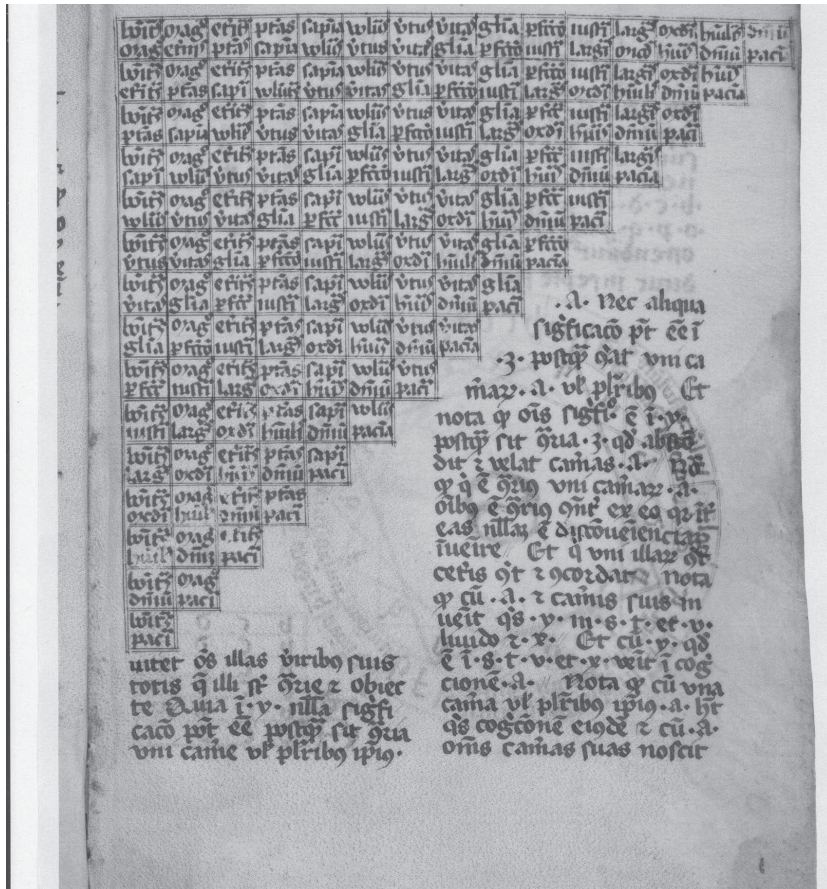
Fig. 7. Vat. lat. 5112, f. 4r Second Figure of Figure A of *ArsComp*

It is another way of showing the same information: the binary relationships between basic concepts of the Lullian system. But the Lullian Art also works with combinations of more than two elements, especially from *ArsInv*. To do this, Llull will use volvelles, that is, circular figures with moving wheels, which constitute hypergraphs with hyperedges that can link more than two vertices. This is the case of the Demonstrative Figure of *ArsDem*, which in the manuscripts has been executed by drawing the discs of the figure, mentally movable, or, more rarely, cutting parchment discs and sewing them through the center.
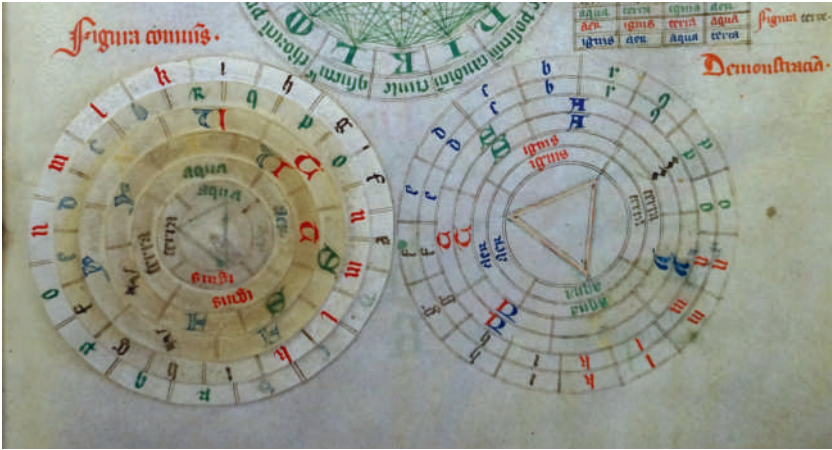
Fig. 8. Nikolaus Hospital, ms. 83, f. 2r Demonstrative Figure in double format: as a volvelle and in two-dimensional representation

In *ArsInv*, Fourth Figure is also a hypergraph in the form of a volvelle and represents ternary combinations of nine letters.

Fig. 9. Uppsala, Universitetsbiblioteket, ms. C51, f. 105r, Fourth Figure

All the possible ternary combinations of the Fourth Figure are resolved explicitly in the Table, with 84 columns of 20 components each, which Llull introduces from his work *Tabula generalis* (1293-1294).

The Elemental Figure stands out in the graphic apparatus of the Art's[23] first versions. It has an appearance of a table made up by four quadrangles – one for each of the four elements – with sixteen compartments. Here the quality of each element has been added between square brackets: fire is hot, earth is dry, water is cold and air is humid.

The Figure of Fire [heat]

fire	air	water	earth
air	fire	earth	water
water	earth	fire	air
earth	water	air	fire

The Figure of Air [moisture]

air	fire	water	earth
fire	air	earth	water
water	earth	air	fire
earth	water	fire	air

The Figure of Water [cold]

water	earth	air	fire
earth	water	fire	air
air	fire	water	earth
fire	air	earth	water

The Figure of Earth [dryness]

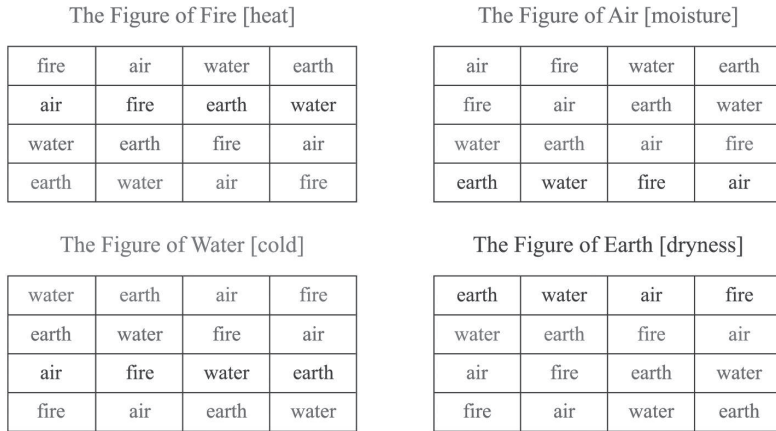| earth	water	air	fire
water	earth	fire	air
air	fire	earth	water
fire	air	water	earth

Fig. 10. Elemental Figure

Gradation, which Llull systematized in his own way, was the way in which medieval science usually measured the intensity of the simple elements present in compounds. If the Elemental Figure has the form of a quadruple table, it is because Llull does not intend to express concepts in terms of their relationship (which would be represented by a graph) but rather to value those relationships as quantitative or qualitative comparisons. In the Lullian theory of elements and intensity degrees of elementary qualities, it will be essential to introduce the concept of "equal or superior" ($\geq$) and therefore the notion of lattice of mathematical set theory.[24]

The interaction of all these graphic elements gives rise to a combinatorial. Llull, in harmony with medieval logic, conceived of his Art as a tool for forging judgments and syllogisms. It was for this purpose that he began to assign letters to various Art concepts in *ArsComp*.

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[23] See for example the Elemental Figure of *ArsComp* in the lower part of illustration 2.

[24] See a detailed study of the Elemental Figure as a lattice in A. BONNER and P. J. PLANAS, "La farmacologia de Ramon Llull: de la medicina en la figura elemental als reticles matemàtics", in *Studia lulliana* 62 (2022), in press.

Later, by means of combinations of letters, he established the necessary relationship between the terms of a judgment or of a certain number of judgments. Llull called this operation "the formation of compartments." From *ArsInv*, the use of letters disappears from the text of the Art, which is based on the definitions of the concepts under discussion and acquires a propositional aspect. All Art was directed towards a specific goal: to discover in any subject area the appropriate terms to form judgments and syllogisms, and to build logical reasoning through necessity.[25] It is here, in this attempt, naive and brilliant at the same time, to mechanize knowledge, where the peculiar power of seduction that the Art of Llull has exerted throughout history originates, from the time of Nicholas of Cusa to that of Leibniz.

The symbolic dimension of graphic representations

Throughout his work, Llull shows little interest in symbolic figurative representations. We are wrong if – as it has sometimes been done – we give a symbolic interpretation to the circularity of Figure A (illustration 5) of the Art, which represents God, because what matters in the diagram is not the circumference that contains the sixteen divine dignities, but the star drawn by the edges that join them to form a mathematical graph; the outer circumference is perfectly expendable.

The figures that we see him developing in the following versions of his Art are not of a symbolic nature, but have a functional nature, they are executed with a formal sobriety that tends to accentuate with their evolution and they do not offer any margin for artistic figuration. In *ArsDem* we will find that Figure V, which represents the set of virtues and vices, has double color, blue and red respectively for virtues and vices, colors with very obvious connotations but with little effective relevance regarding their operation.[26]

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[25] For a study of the Art and the evolution of its components see Bonner, *The Art* and now, the supplement to this manual, Bonner "Supplement".

[26] For the use of color in the works of Ramon Llull, see L. Badia, "Natura i semblança del color a l'opus lul·lià: una aproximació", in *Studia lulliana* 43 (2003), 3-38. On the other hand, among the oldest Lullian manuscripts, only in the Munich, Bayerische Staatsbibliothek, Clm. 18446 the figures of *Ars compendiosa* (1299) appear richly decorated and illuminated; and only in one of them do we find a certain symbolic representation: the Fourth Figure of the Art, a volvelle to which reference has already been made, contains the figure of a little man in an attitude of moving the central wheel.
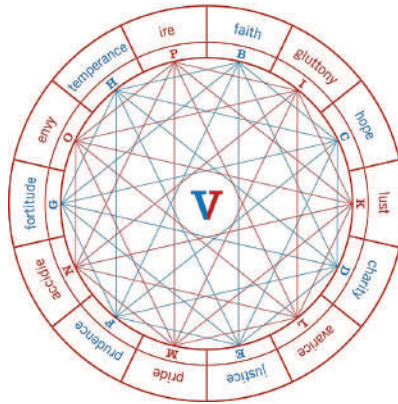
Fig. 11. Figure V *ArsDem*

The figures of Art are not, in any sense, an illustration of a discourse, they do not embody a visual or symbolic representation of any reality, neither are they representations of any specific thought. On the contrary, they are only instruments that the reader must use to generate scientific or contemplative arguments. We could say that they are something to the extent that they are put into operation by someone, the Art's user. Therein lies its diagrammatic character.

In works that are not strictly artistic, but in derivations or applications of Art, or even in works prior to Art, we find another type of figure with a structuring function of discourse and, at the same time, also a symbolic one: tree diagrams. Perhaps the oldest and clearest example of this dimension is offered by the *Llibre de contemplació*, in book IV, which is divided into six distinctions (XXXIII-XXXVIII), each of which deals with a different, schematically represented, tree. In total, then, six trees. For example, in chapter 255, distinction XXXVII, there is a "Tree of the Ten Commandments." In the copy of *Llibre de contemplació* executed by Guillem Pagès, of whom we have already spoken, completed in July 1280 – the oldest dated Lullian codex, very close to Llull's lifetime –, the tree looks like this:

In reality, it is an anecdotal illustration that in no way contributes to the functionality of the figure and, furthermore, it must be taken into account that this codex, despite being contemporary, cannot be directly linked to Llull.
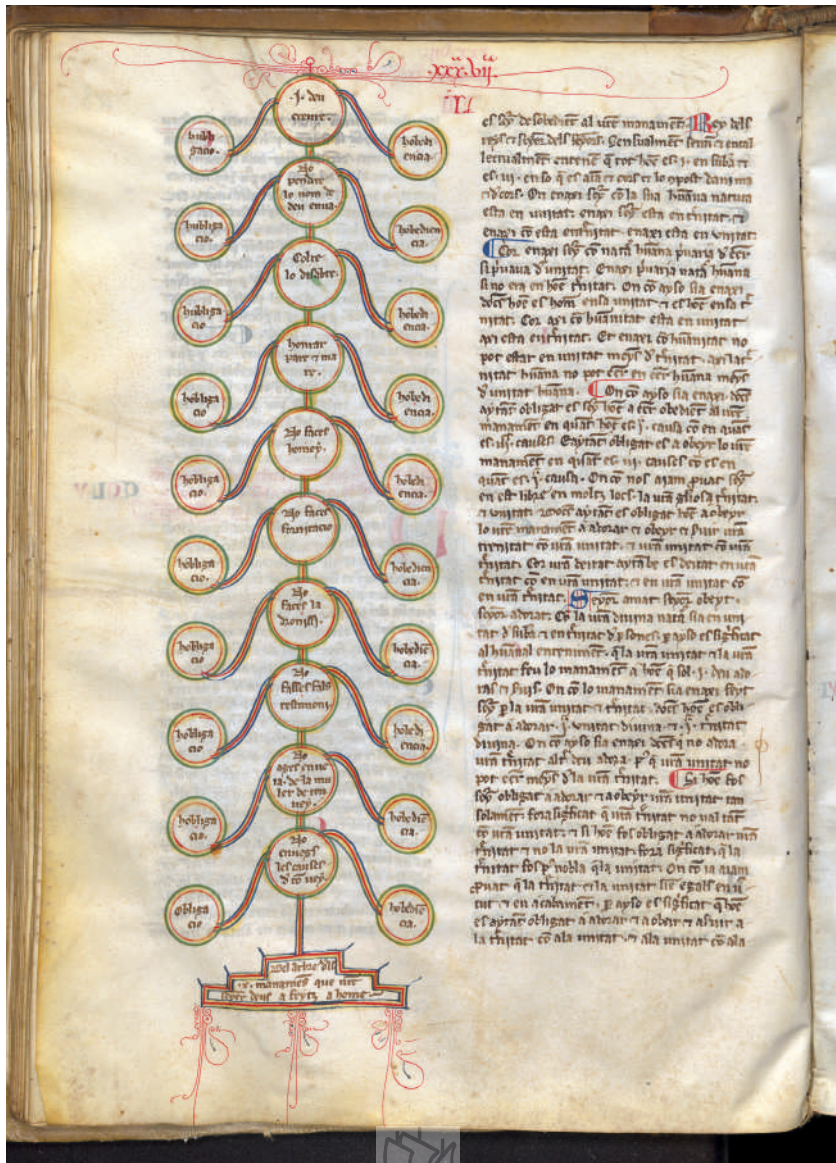
Fig. 12. Milan, Biblioteca Ambrosiana, ms. D 549 lower, f. 304bisv,
"Tree of the Ten Commandments" from the *Llibre de contemplació*
(ch. 255)

Each of the Ten Commandments appears in descending order on the tree's trunk and each one is related to the concepts of 'obligation' and 'obedience' that appear on the leaf that springs from each Command-ment from left to right of the trunk. In other words, it is being indicated that the concepts of 'obligation' and 'obedience' will be present in each chapter of the distinction, which will deal with each Commandment. It is a visual outline of the chapter's contents. The same relationship that is established between tree's concepts and Commandments could be expressed diagrammatically as follows:
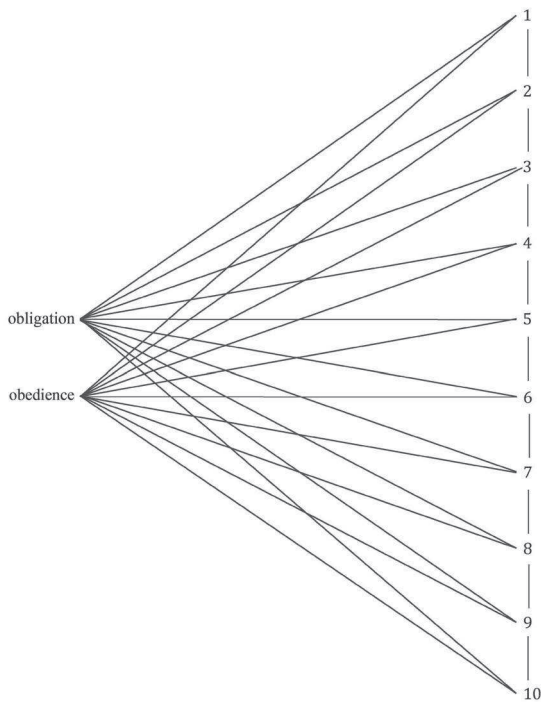

Fig. 13. Diagrammatic representation of the content of the "Tree of the Ten Commandments" from the *Llibre de contemplació* (ch. 255)

Strictly speaking, it was not necessary for Llull to draw a tree in order to visualize the relationship between the components of the XXXVII distinction. If he does it, it is because the tree is a figure sanctioned by the iconographic tradition that allows a diagram to be embellished figuratively and symbolically.

In the previous distinction, XXXVI, of the *Llibre de contemplació*, chap. 238, the "Tree of faith and reason" is represented in the figure of the cross. The figure gives rise to a singular page in which the text accommodates itself to the shape of the cross. The enumeration of the exhibition contents is noted in a rubric on the arms of the cross, with the peculiarity that the rubrics that go on the arms of the crossbar have been copied vertically.

Fig. 14. Ambrosiana, ms. D 549 lower, f. 278v, "Tree of faith and reason in the figure of the cross" from the *Llibre de contemplació* (ch. 238)

Unlike the "Tree of the Ten Commandments," this figure does not interrelate the concepts, but simply lists them. It is, then, a visual and emblematic index of the distinction's contents. The idea of tree is associated with the symbol of the cross and this endorses the name of the figure. Finally, the figure seems to be justified implicitly (but not in an open way) by the theme of the XXXVI distinction: it could suggest that the presentation of the fourteen articles of the Christian faith that is developed in chapters 245-254 has its symbolic foundation in the mystery of the cross.[27]

Graphic images and verbal descriptions: Lullian diagrammatic reasoning as a mental activity

The graphic images materially grasped in the codices are not the only type of images that Llull displays in his works. Textual descriptions of images, the procedure that classical rhetoric knows as *ekphrasis* – ἔκφρασις, *descriptio* in Latin treatises –, also play a fundamental role in Lullian diagrammatics. In Llull's opus we find numerous examples of ekphrastic speeches. For example, in chapters 352-357 of the *Llibre de contemplació*, Llull develops a particular method of contemplation inspired by the four meanings of Scripture in biblical exegesis. Narrative texts that exemplify a new theory of allegory present startling and dazzling imagery that could have been the subject of fanciful symbolic illustrations. However, Ramon chooses to develop a completely discursive description so that the reader has to form a mental idea of what is exposed.[28]

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[27] For a detailed study of the presence of graphic elements in the six manuscripts copied by Guillem Pagès, a scribe in Llull's service to whom I have already referred, see now G. Pomaro, "Dall'ars scripta all'ars figurata: Raimondo Lullo (1232-1316) e la tradizione figurata dell'ars combinatoria", in *Scribes and the Presentation of Texts (from Antiquity to c. 1550): Proceedings of the 20th Colloquium of the Comité international de paleographie latine. Beinecke Rare Book & Manuscript Library, Yale University (New Haven, September 6-8, 2017)*, ed. B. A. Shailor and C. W. Dutschke, Turnhout 2021, 411-437.

[28] The most prominent is the contrast between body and soul in chapter 354, which features three maidens and a two-headed beast, rising and falling on the ends of a scale. See J. E. Rubio, "A capítol en l'ús de l'al·legoria en Ramon Llull: exegesi del capítol 354 del *Llibre de contemplació*", in *Studia lulliana* 47 (2007), 5-27 for an analysis of this chapter.

In works derived from Art, but not strictly artistic, we find detailed descriptions of figures in their absence. This is the case of *Arbre de ciència* [*Tree of Science*] (1295-1296), which is the work that presents Llull's vision of the world in more detail. Fourteen trees make up the "Tree of Science" complex: from the elemental and vegetable trees to those of Jesus Christ and God. To these, two instrumental trees are added, that of examples and that of questions.[29] Thus, we have sixteen trees that make up a single "Tree of science." Each of those trees is described in detail in the text but not graphically represented in any of the older testimonies. The ekphrastic discourse exposed is the most elaborate found in Llull's opus and gradually builds the mental image of the Tree of trees, which is impossible to represent bidimensionally, since it is a tridimensional object: a tree in which multiple tree structures come together, blurring into their unity to become part of a whole. The use of ekphrasis allows one to mentally evoke the character of Llull's ontology: a unified body with all its parts interrelated in an orderly manner. Furthermore, Llull does not want to compile an encyclopedia in which the diversity of the world is presented in an orderly manner, separating its parts; instead, he intends to show simultaneously the indissoluble unity of the world in its constitutive diversity.[30]

The use of ekphrasis also affects the figures of the Art because sometimes the diagrams are presented without any description, as in *ArsComp*; in others, with a displaying of figures and its functioning into the image, as in *ArsDem* or *ArsInv*. The description of figures concerning their graphic representation entails an interaction between text and image. Llull himself expresses it at the beginning of the tree figure's description in the *Liber principiorum medicinae*: "Naturale quidem est intellectui, ut ipse per demonstrationem factam per uisum et

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[29] For the structure of the work, see A. Bonner, "The structure of the Arbor scientiae", in *Arbor Scientiae: der Baum des Wissens von Ramon Lull. Akten des Internationalen Kongresses aus Anlass des 40-jährigen Jubiläums des Raimundus-Lullus-Instituts der Universität Freiburg i. Br.* (Instrumenta Patristica et Mediaevalia. Subsidia Lulliana 1) ed. F. Domínguez Reboiras, P. Villalba Varneda and P. Walter, Turnhout 2002, 21-34.

[30] For the *Arbre de ciència* as an ekphrastic discourse, see A. Bonner and A. Soler, "La representació de l'arbre en l'*Arbre de ciència* de Ramon Llull", in *Imago* 8 (2016), 131-142, and A. Bonner and A. Soler, "Representació gràfica i ècfrasi en l'obra de Ramon Llull", *Magnificat Cultura i Literatura Medievals* 3 (2016), 74-80.

per auditum melius intelligat, quam per auditum solum."[31] For Ramon, the conjunction of both presentations, verbal and graphic, is preferable. In this same work, a very detailed tree's description is provided, which is depicted in many manuscripts, one of which corresponds to a manuscript copied by Guillem Pagès during the author's lifetime. Llull states the relevance for readers of the tree's graphic image, something that he expressly recommends at the end of the description:

> Quicumque autem hanc artem intelligere et scire desiderat, oportet ipsum scire perfecte alphabetum superius positum et ipsum in memoria semper retinere; et etiam oportet ipsum facere flores alios similes floribus dictae arboris fabricatos et mobiles, ut alios ipsorum florum cum aliis commiscere et formare demostratiue possit, necnon ipsos augmentare et diminuere ac etiam adaequare ad declarandum, qualiter medicus suas debet componere medicinas.[32]

This recommendation to build a mobile figure for personal use, based on the figure described, constitutes an invitation to develop an autonomous work with the graphic resources and the possibilities offered by them. In any case, the verbal description of the diagrams, whether long or brief, underlines their importance, so the reader does not overlook that they are an essential instrument for organizing and codifying the foundations of the entire combinatorial apparatus of the system: they are the elements that articulate the diagrammatic reasoning of the Art. The assimilation of the figure's morphology and their operations represented by material and mental visualization looks for the rational soul elevation from intellectual knowledge to spiritual contemplation. Thus, he declares it in *Ars amativa*:

> Ratio quidem, quare figuras tradimus in hac Arte consistit, ut in eis fiat principiorum huius Artis speculatio; sensuali namque speculatione extollitur intellectus ad speculandum intellectualia, et inde uoluntas ad ea diligendum.[33]

Both in their plastic and verbal dimension, the figures are a tool that helps meditation and contemplation, in the intellectual as well as in the

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[31] Raimundus Lullus, *Liber principiorum medicinae*, in *Quattuor Libri Principiorum*, ed. M. A. Sánchez Manzano (CCLM 185), Turnhout 2006, 438-439. For the use of ekphrasis in the works of Ramon Llull, see Bonner and Soler "Representació gràfica".

[32] Raimundus Lullus, *Liber principiorum medicinae*, 445.

[33] Raimundus Lullus, *Ars amativa boni et Quaestiones quas quaesivit quidam frater minor*, ed. M. M. M. Romano y F. Santi (CCCM 183), Turnhout 2004, 124.

spiritual sphere. The Lullian Art gathers structure and systematization to this exercise.

Ramon proposes an unavoidable reader's commitment in the process of elevating the soul, which can take various forms throughout his work. Sometimes it is about posing an apparent darkness of the proposed discourse as an intellectual challenge to overcome.[34] Others, it is about leaving the end of a work open, with no apparent solution, so that the reader should find it, as the *Llibre del gentil* referred above. Or even proposing a remarkable apparatus of questions and answers to assimilate a certain version of the Art. This last resource is part of a more general program that he calls "habituation", which consists not only in finding solutions to questions, but in formulating and solving new questions that the reader must place on his own. As he says in the penultimate section of *Ars brevis* (1308):

> Haec pars est de habituatione huius Artis. Et ipsa diuiditur in tres partes. Quarum prima est de tredecim partibus, in quas haec Ars diuiditur. Et illas artista huius Artis debet habituare, ut ipse sciat applicare quaestionem ad illum locum seu loca, quaestioni dispositum seu disposita secundum proportionem materiae quaestionis.

> Secunda pars est, quod habituet modum et processum textus huius Artis, tenendo modum textus ad probandum et soluendum quaestiones peregrinas illo modo, per quem sunt explicatae in textu: sicut unum exemplum, per quod aliud est declaratum et exemplificatum. Tertia pars est, quod ipse habeat modum multiplicandi quaestiones et solutiones ad unam et eandem conclusionem [...][35]

Llull looks forward reader to make figures work that express the essential connectivity of the system, so that he develops dynamic and open knowledge processes, in which anyone could participate in the act of finding (*invenire*) questions and formulating new answers (*demonstratio*). It is precisely in this inventive dimension that Llull's figures are sometimes not merely a mnemonic aid. Definitely, in this sense, Lullian Art is not in the manuscripts, in the graphic figures materially reproduced in books or in the expositions describing them, but in the mind of those who have assimilated it and reason in accordance

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[34] J. SANTANACH "Ramon Llull i l'obscuritat que il·lumina. Apunts sobre l'origen i rendibilitat literària d'un recurs exegètic", in *Anuario de Estudios Medievales* 45,1 (2015), 331-354.

[35] RAIMUNDUS LULLUS, *Ars brevis*, ed. A. MADRE (CCCM 38), Turnhout 1984, 253.

with it. This is an issue about diagrammatic reasoning, an intellectual framework made for adopting visual resources and, therefore, a mental activity above all.

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