What is the Teichmüller space of a holomorphic function?

A HOLODYN seminar series

- Universal coverings, Fuchsian gropus, and the hyperbolic metric **(1-2 lectures)**
  (1) General facts about universal coverings
    i. Universal coverings (Donaldson, p. 46–47)
    ii. Deck transformations and fundamental groups (Abate, p. 38–39)
    iii. Lifts (Abate, p. 40–42)
  (2) The hyperbolic metric and its simply connected models (Hubbard, p. 23–31)
    i. Definition and models
    ii. Geodesics
    iii. Classification of isometries
  (3) The hyperbolic metric in multiply connected Riemann surfaces (Hubbard, Prop. 3.2.1 and p. 69, 71, 73)
    i. Definition
    ii. Hyperbolic distortion and the Schwarz–Pick lemmma
    iii. Geodesics
    iv. Doubly connected Riemann surfaces
  (4) Fuchsian groups (Hubbard, p. 75–76)
    i. Deck transformations as Fuchsian groups
    ii. Limit sets
    iii. Geometric convergence (Matsuzaki and Taniguchi, p. 205)
- Quasiconformal isotopies **(1 lecture)**
  (1) The ideal boundary (Hubbard, p. 87–88)
  (2) Characterising quasiconformal isotopies (Earle and McMullen, Thm. 1.1 and Cor. 2.4)
- The Teichmüller space of a Riemann surface **(1 lecture)**
  (1) Two equivalent definitions (de Faria and de Melo, p. 173–174; McMullen and Sullivan, p. 361–362)
  (2) Finite-type Riemann surfaces and the Fenchel–Nielsen coordinates (Buser, p. 27–29)
  (3) Teichmüller space vs moduli space: the annulus and the torus (Hubbard, Prop. 3.3.7; Donaldson, p. 93–95)
- The Teichmüller space of a holomorphic function **(≥ 2 lectures)** (McMullen and Sullivan, p. 363–367 and 370–379)
  (1) Holomorphic relations
  (2) Definition
  (3) Foliated annuli
  (4) Covering relations and closed subgroups of $\mathbb{H}$
  (5) The case of rational maps
  (6) The modular group and the covering of moduli space

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