Combinatorics of associativity, toric varieties, and wonderful models

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In 1960s, Jim Stasheff defined associahedra, remarkable CW-complexes that can encode a homotopically coherent notion of associativity. They have been realised as polytopes with integer coordinates in several different ways over the past few decades. I shall explain that the realisations of associahedra due to Loday lead to toric varieties of particular merit. These varieties have been already studied by Escobar under the name "brick manifolds", in the context of subword complexes for Coxeter groups. It turns out that they also arise as "wonderful models" in the sense of de Concini and Procesi for certain subspace arrangements. Guided by that geometric picture, I shall argue that in some sense these varieties give a "noncommutative version" of Deligne-Mumford compactifications of moduli spaces of genus zero curves with marked points, in particular they give rise to remarkable algebraic structures resembling cohomological field theories of Kontsevich and Manin. This is a joint work with Sergey Shadrin and Bruno Vallette.