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The height of toric subvarieties

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This is a report on joint work with P. Philippon (Paris) and J.I. Burgos (Barcelona). A complete toric variety X of dimension n is determined by a lattice N and a complete lattice fan Σ on $N_{\mathbb{R}}$. This variety is naturally equipped with the action of a complex torus. An equivariant ample line bundle L on X determines a lattice polytope Δ in $M_{\mathbb{R}} := N_{\mathbb{R}}^{\vee}$. Most algebro-geometric properties of the pair (X, L) can be easily read off from this polytope Δ.
The exponential map determines a parametrization of the open orbit X_0 by $N_{\mathbb{C}}$. Assume that L is equipped with a positive Hermitian metric that is invariant under the action of the compact torus. The logarithm of the norm of a section of L gives then a strictly convex function f on $N_{\mathbb{R}}$. The stability set of this function is the polytope Δ and its Legendre dual $f^{\vee}$ is a strictly convex function on Δ. This is the symplectic potential of the Kahler structure of X_{Σ} in the Guillemin-Abreu theory.
We prove that the height of X with respect to the metrized line bundle $\overline{L}$ is given by $(n+1)!$ times the integral of $-f^{\vee}$ with respect to the normalized Haar measure of $M_{\mathbb{R}}$. This is the arithmetic analog of the expression of the degree of X as $n!$ times the volume of the polytope.