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A Matroidal Approach to Bounding Hilbert functions and Betti numbers of Fat Points in $\mathbb{P}^2$

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By work of Macaulay and Geramita et al., it is known exactly which functions occur as Hilbert functions of radical ideals defining points in projective space. A recent paper by Geramita- Migliore-Sabourin (J. Alg., 2006) raises the question of which functions occur as Hilbert functions of symbolic squares of such ideals for points in $\mathbb{P}^2$ (i.e., of ideals of double points). Using basic double linkage methods, for each function f which is the Hilbert function of a set of reduced points in $\mathcal{P}^2$, GMS finds a set of points $\{p_1, ..., p_r\}$ whose ideal I has Hilbert function f and for which they are able to compute the Hilbert function g of the symbolic square $I^{(2)}$. They also give a criterion for when g is uniquely determined.
I will discuss joint work with Susan Cooper and Zach Teitler taking a different approach to this problem. We associate a matroid to each set of points in $\mathbb{P}^2$. The matroid can be regarded as the incidence matrix of all collinear subsets of the points (i.e., a matrix whose nonzero entries are all 1s, where each row corresponds to a line through two or more of the points, and a 1 entry in column i of the row indicates the point p_i lies on the line). Given any subset of the rows of the matrix and given arbitrary multiplicities m_i for a fat point scheme $Z = m_1p_1 + ... + m_rp_r$, we give upper and lower bounds for the Hilbert function of the ideal $I(Z)$ of Z. We also give a criterion for when these bounds coincide, generalizing the criterion of GMS. In addition, when this criterion is satisfied, we also give bounds on the graded Betti numbers of $I(Z)$.
