PARTIAL DIFFERENTIAL EQUATIONS

XAVIER ROS OTON, FALL 2024

Evaluation

The final grade of the course will be

50% Problem lists + 50% Final presentation

- **Problem lists**: Each of the five Chapters has a Problem set, available at

`www.ub.edu/pde/xros/UB.html`

For each of these problem sets you have to solve exactly 3 problems and send them to Marvin Weidner (mweidner@ub.edu) before the corresponding problem session:

- Chapter 1: October 4th
- Chapter 2: October 22nd
- Chapter 3: November 12th
- Chapter 4: November 22nd
- Chapter 5: December 13th

Alternatively, you can work in groups of 2 or 3 people, and in that case you have to solve exactly 6 problems. Your grade is then half of the total points for the 6 problems you solved.

Notice that you can have more points if you solve the difficult problems than if you solve the easier ones.

If you do not send the problems before the corresponding class, you will have 0 points for that Chapter.

- **Final presentation**: You will have to choose one of the Chapters 2, 3, 4, or 5, and prepare it in order to make a presentation in January. After the presentation, we will ask questions about such Chapter (and how it relates to other parts of the course).

The maximum grade if you prepare one Chapter will be 9 (out of 10). If you want more than 9, you have to prepare at least two Chapters.

Syllabus

1. **Overview and preliminaries**
   - Overview of (linear and nonlinear) PDE
   - Sobolev spaces and compactness
2. **Laplace equation and eigenfunctions**
   - Existence of solutions to the Dirichlet problem
   - Eigenfunctions of the Laplacian
   - Regularity of solutions
3. **Linear evolutionary PDE**
   - Heat equation
   - Wave equation
   - Schrödinger equation
4. **Nonlinear parabolic PDE and the Navier-Stokes equations**
   - Short-time existence for nonlinear parabolic PDE
   - Uniqueness and qualitative properties of solutions
   - Navier-Stokes equations and the Millenium Prize Problem
5. **Nonlinear elliptic PDE and the Calculus of Variations**
   - Introduction to the calculus of variations
   - Minimizers of convex functionals, Hilbert 19th and 20th problems
   - Semilinear equations

Prerequisites: A solid background of Real Analysis is required.
Some basic knowledge of PDE and/or Functional Analysis is useful but not necessary.